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# Ground effects on pressure fluctuations in the atmospheric boundary layer

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Proposals are made for modelling the pressure-containing correlations which appear in the transport equations for Reynolds stress and heat flux in a simple way which accounts for gravitational effects and the modification of the fluctuating pressure field by the presence of a wall. The predicted changes in structure are shown to agree with Young's (1975) measurements in a free stratified shear flow and with the Kansas data on the atmospheric surface layer.

## 1. Introduction

The effects of the earth's gravitational field on turbulence have received much attention both from those interested in the atmospheric boundary layer and from those concerned with the dispersal of jets or plumes into the environment. In both types of flow gravitational influences may be profound. It does not seem to have been fully recognized, however, that *the effects are fundamentally different in the two types of flow, particularly under stable conditions.* The differences are illustrated in table 1, which gives the variation of three dimensionless turbulence parameters under stable stratification. The two flows compared are the lower region of the atmospheric boundary layer and a horizontal, nominally homogeneous free shear flow in which a linear vertical profile of mean velocity and temperature has been established. The experimental data on the atmospheric surface layer are from the Kansas measurements (Haugen, Kaimal & Bradley 1971; Businger *et al.* 1971; Wyngaard, Coté & Izumi 1971) while the entries for the free shear flow relate to the measurements of Webster (1964) and Young (1975). Both flows are close to local equilibrium, energy generation by the combined effects of mean shear and buoyancy being approximately balanced by viscous dissipation. For the free shear flow, the variations brought about by an increasingly stable stratification are, for the most part, what could be inferred by considering the action of buoyant generation in the conservation equations for the turbulent stresses and heat fluxes. For example, in the equation for $\overline{u_3^2}$, the mean-square vertical velocity fluctuation, there is a damping due to buoyancy equal to $2R_f$ times the generation rate of kinetic energy due to shear, $R_f$ denoting the flux Richardson number. Buoyant generation is absent, however, from the equation for streamwise fluctuations. It is to be expected, therefore, that $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ would progressively decrease in an increasingly stable stratification. Precisely parallel remarks may be made about

| Turbulence parameter | Changes produced by increasing $R_f$ from 0 to 0·2: Homogeneous free shear flow | Changes produced by increasing $R_f$ from 0 to 0·2: Atmospheric boundary layer (Kansas experiments) |
|---|---|---|
| $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ | Decreases by about 30 % | Increases by about 20 % |
| $\overline{u_3\gamma}/\overline{u_1\gamma}$ | Decreases by about 70 % | Increases rapidly by 50 % then levels out |
| $K_H/K_M$ | Decreases by about 50 % | Initially decreases by 10 % then rises slowly |

TABLE 1. Effects of stable stratification on atmospheric boundary layer and free shear flow.

the behaviour of the ratio $\overline{u_3\gamma}/\overline{u_1\gamma}$ of the vertical to the horizontal heat flux. We note, however, that this 'natural' direction of variation is not observed in the atmospheric boundary layer. The ratio $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ actually appears to rise with increasing stability (see Arya's (1972) representation of the Kansas data, figure 1*a*) while the heat-flux ratio first falls sharply and then levels out to a nearly uniform value. As seen from table 1, a strikingly dissimilar variation is also observed in the ratio $K_H/K_M$ of the turbulent diffusivities of heat and momentum.

Of course, measurements in buoyancy-affected turbulence are not easy to make and the question must arise as to whether some of the reported experimental trends are anomalous. A single difference in trend between the free shear flow and the boundary layer could perhaps be attributed to measurement error. The behaviour summarized in table 1, however, is basically and consistently different for the two situations. One is therefore led to conclude that the buoyant effects in the two flows are indeed fundamentally different.

We can unambiguously identify the cause of the differing behaviour if we accept that, in both kinds of flow, the fine-scale motion is isotropic and that conditions are sufficiently close to local equilibrium for transport effects to have only a minor influence. In these circumstances the only correlations in the conservation equations for the Reynolds-stress and heat-flux components are those containing pressure fluctuations. The differences between the two classes of flow are thus due to the direct and indirect influences of the ground on these correlations.

Many workers, beginning with Monin (1965), have used the transport equations for the Reynolds stresses and heat fluxes as the starting point for modelling buoyant effects on turbulence. This level of closure is a particularly attractive one for modelling the effects of gravity because the terms representing both the direct and the indirect effects of the force field on the Reynolds stresses and heat fluxes appear naturally in the governing equations. There is thus the hope that a closure in which the empirical inputs are chosen by reference to non-buoyant flows may be extrapolated to predict the effects of varying degrees of stratification on the shear flow. This has been the underlying rationale of all second-order closures for buoyant flow from Monin (1965) and Ellison (1966, unpublished note cited by Yaglom 1969) onwards. The approach has been at least partly successful. Indeed, the present work indicates that such shortcomings as earlier models did possess are due mainly to an inadequate modelling of *neutral* shear flows rather than to the extrapolation principle itself.

In order to close the exact Reynolds-stress and heat-flux equations, approximations are needed for the correlations between the pressure and strain fluctuations (called 'the pressure–strain correlations', $\phi_{ij}$) in the Reynolds-stress equation and the corresponding pressure–temperature-gradient correlations $\phi_{i\gamma}$ in the heat-flux equation. Rotta's (1951) pioneering paper on closing the Reynolds-stress equations in non-buoyant flows had shown that there were two agencies contributing to $\phi_{ij}$: one due to purely turbulent interactions and the other arising from shear in the mean velocity. Most subsequent proposals neglected the influence of the mean strain, however. Here we may cite the contributions of Ellison (1966; see Yaglom 1969), Donaldson and his colleagues (Donaldson, Sullivan & Rosenbaum 1972; Lewellen 1975) and the earlier work by Lumley and his associates (Lumley 1972; Wyngaard & Coté 1974). Mellor (1973) included a primitive model of $\phi_{ij}$ but neglected entirely mean-strain effects on $\phi_{i\gamma}$. The resultant model reproduced some of the features of the atmospheric layer but the normal-stress ratios displayed an opposite variation from measurements (predictions of the ratios of heat fluxes and eddy diffusivities were not shown). More recently Launder (1975*a*) has shown that, in addition to mean-strain influences, there are important buoyant contributions to $\phi_{ij}$ and $\phi_{i\gamma}$; models of these processes, exact in the case of isotropic turbulence, have been reported by Launder (1975*b*) and Lumley (1975).

Nearly all the proposals mentioned above were made without regard to the presence of a wall. Only those of Launder (1975*a*, *b*) and Lumley (1975) were specifically limited to free shear flows. Although Monin (1965) recognized that the ground itself exerted an effect on $\phi_{ij}$ and $\phi_{i\gamma}$, it was not until the rather neglected contribution of Shir (1973) that a physically consistent wall-effect contribution to $\phi_{ij}$ appeared. Shir, extending the suggestions of Daly & Harlow (1970), supposed that the wall correction was needed only for the 'turbulence' part of $\phi_{ij}$. Irwin's (1973) analysis of the exact pressure-fluctuation equation (for non-buoyant flows) suggested, however, that both the mean-strain and the turbulence contribution would be affected yet, curiously, only the former was included in his closure. Launder, Reece & Rodi (1975) included near-wall corrections for both the mean-strain and the turbulence part which allowed the prediction of several types of boundary-layer and free shear flows. That work and that of Irwin & Arnot Smith (1975) have shown that the inclusion of the near-wall effect on $\phi_{ij}$ is crucial to predicting the great sensitivity of boundary layers to wall curvature. In a rather different connexion Reece (1977) has demonstrated that the introduction of a near-wall correction greatly improves the prediction of turbulence-induced secondary flow in square-sectioned ducts. Thus, in summary, several studies in isothermal shear flows have found that the recognition of the wall's influence on $\phi_{ij}$ helps to explain a number of hitherto unresolved features of wall shear flows.

In most of the above work the strength of the wall effect on $\phi_{ij}$ was assumed to depend on the ratio $l/x_n$ of a turbulence length scale to the distance from the wall. Where $l$ increases linearly with $x_n$ the strength of the term was thus uniform; as the length scale levelled off in the outer part of the shear flow, wall influences diminished.

The present paper has sprung from the realization that the use of a wall correction similar in form to that already used in non-buoyant flows might account for the paradoxical behaviour indicated in table 1. Experiments by several workers (see, for example, the monograph by Turner 1973) have shown that in the atmospheric boundary layer the ratio $l/x_n$ is extremely sensitive to the strength of the stratification.

As the Richardson number progressively increases from zero $l/x_n$ falls dramatically; this in turn (so our reasoning went) will reduce the significance of the near-wall correction. Now, the effects of a wall on non-buoyant flow are, in many respects, similar to those of a stable gravitational field on a free shear flow (diminishing the level of vertical fluctuations, reducing the ratio of shear stress to turbulence energy, etc.). In the non-buoyant free shear flow the normal-stress ratio $\overline{u_3^2}/\overline{u_1^2}$ has a value of approximately 0·5 while a typical value in the uniform-stress layer of a wall flow would be about 0·2. The effects of stable stratification on the wall flow are twofold: direct damping of the vertical fluctuations is combined with a progressive weakening of the wall effect as stability is increased. It is our contention that in weakly stratified flow the second effect predominates, so that $\overline{u_3^2}/\overline{u_1^2}$ *increases* with increasing stability towards the corresponding free-flow value. In strongly stable flow the characteristic eddy size is determined entirely by the stability and is quite independent of distance from the wall. The turbulence behaves as in a free flow and any further increase in stability reduces the levels of $\overline{u_3^2}$ and the shear stress until, at a critical flux Richardson number of around 0·25, the turbulence disappears and is replaced by gravity waves. The behaviour indicated in table 1 and, to anticipate, in figure 1 (*a*) may plausibly be attributed to the diminishing influence of wall damping as stability is increased.

In the present contribution we have attempted to test the correctness of this basic conjecture. The turbulence model adopted is, in its main features, the same as that developed in our earlier work on free shear flows (Launder 1975*a*; Gibson & Launder 1976). To this we have added a wall correction obtained by generalizing Shir's (1973) proposal. The predicted results do indeed show the strongly different behaviour of stratification in wall-affected and free shear flows. Most of the effects shown in table 1 are correctly predicted by the model though the sharp fall in the ratio of horizontal to vertical heat flux for weakly stable flow is not recovered.

The critical flux Richardson number at which the turbulence collapses is predicted as 0·25 but the calculations do not include any contribution from the wave motion. However, the collapse is so rapid that contributions from this source may reasonably be assumed to be insignificant up to Richardson numbers only just below the critical value.

## 2. The Reynolds-stress and heat-flux equations

For a high Reynolds number (locally isotropic) shear flow in local equilibrium, the exact equations governing the level of the Reynolds stresses and heat fluxes may be written as

$$P_{ij} + G_{ij} - \tfrac{2}{3}\delta_{ij}\epsilon + \phi_{ij} = 0, \tag{1}$$

$$-\overline{u_i u_k}\,\partial\Gamma/\partial x_k + P_{i\gamma} + G_{i\gamma} + \phi_{i\gamma} = 0. \tag{2}$$

The quantities $P_{ij}$, $G_{ij}$, $P_{i\gamma}$ and $G_{i\gamma}$ represent respectively the production rates of $\overline{u_i u_j}$ and $\overline{u_i \gamma}$ attributable to mean shear and buoyancy:

$$P_{ij} \equiv -\{\overline{u_i u_k}\,\partial U_j/\partial x_k + \overline{u_j u_k}\,\partial U_i/\partial x_k\}, \tag{3}$$

$$G_{ij} \equiv -(\alpha/\Gamma)\{g_j \overline{u_i \gamma} + g_i \overline{u_j \gamma}\}, \tag{4}$$

$$P_{i\gamma} \equiv -\overline{u_k \gamma}\,\partial U_i/\partial x_k, \tag{5}$$

$$G_{i\gamma} \equiv -(\alpha g_i/\Gamma)\,\overline{\gamma^2}. \tag{6}$$

Upper and lower case $u$'s and $\gamma$'s denote mean and fluctuating velocities and temperatures respectively, $\mathbf{g}$ is the gravitational acceleration vector, $\alpha$ the dimensionless volumetric-expansion coefficient of the fluid and $\epsilon$ stands for the dissipation rate of turbulence kinetic energy.†

The symbols $\phi_{ij}$ and $\phi_{i\gamma}$ denote respectively the pressure–strain and pressure–temperature-gradient correlations:

$$\phi_{i\gamma} \equiv \overline{\frac{p}{\rho}\frac{\partial \gamma}{\partial x_i}}, \quad \phi_{ij} \equiv \overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}. \tag{7}$$

Apart from the mean-square temperature variance appearing in $G_{i\gamma}$, these pressure-containing correlations are the only unknowns in (1) and (2). Their approximation is thus crucial to the predicted characteristics of the stresses and heat fluxes in a gravitational field.

The pressure fluctuations in (7) are governed by the Poisson equation obtained by taking the divergence of the equation for the fluctuating velocity $u_i$:

$$-\frac{1}{\rho}\frac{\partial^2 p}{\partial x_i^2} = \frac{\partial^2[u_i u_j - \overline{u_i u_j}]}{\partial x_j \partial x_i} + 2\frac{\partial U_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} + \frac{\alpha g_i}{\Gamma}\frac{\partial \gamma}{\partial x_i}. \tag{8}$$

From (8) it is seen that pressure fluctuations arise from three agencies: turbulence interactions, mean-strain effects and density fluctuations in the gravitational term. These contributions produce corresponding effects on $\phi_{ij}$ and $\phi_{i\gamma}$. Moreover, for the class of flows under study here, the presence of the ground modifies the fluctuating pressure field. As we have noted in § 1, it is the ground effect which is responsible for the qualitatively different effects of buoyancy observed in the earth's boundary layer and in free shear flows.

To start with consider the case of turbulence sufficiently remote from a bounding wall for the wall's influence to be negligible. It appears consistent with observations to assume that the action of the fluctuating pressure field will be to make the turbulent velocity and temperature fields less directionally biased, more isotropic. We note further, from (7), that any model of $\phi_{ij}$ must be a second-rank symmetric tensor with zero trace. Following Launder (1975*a*) we adopt the following simple linear model which possesses the directional-blurring tendency noted above:

$$\phi_{ij} \equiv \phi_{ij,1} + \phi_{ij,2} + \phi_{ij,3},$$

with

$$\phi_{ij,1} = -C_1(\epsilon/k)\,(\overline{u_i u_j} - \tfrac{2}{3}\delta_{ij}k), \tag{9}$$

$$\phi_{ij,2} = -C_2(P_{ij} - \tfrac{2}{3}\delta_{ij}P), \tag{10}$$

$$\phi_{ij,3} = -C_3(G_{ij} - \tfrac{2}{3}\delta_{ij}G), \tag{11}$$

where $P$ and $G$ are the production rates of turbulence energy due to the actions of mean shear and buoyancy:

$$P \equiv -\overline{u_i u_k}\,\partial U_i/\partial x_k, \tag{12}$$

$$G \equiv -(\alpha g_i/\Gamma)\,\overline{u_i \gamma}. \tag{13}$$

Equations (9), (10) and (11) correspond to the turbulence, mean-strain and buoyan contributions to the pressure field in (8). In fact, Lumley (1975) and Launder (1975*b*)

† Since we here consider only the local-equilibrium model $\epsilon = \frac{1}{2}(P_{ii} + G_{ii})$.

have shown independently that for isotropic turbulence $C_3$ must equal 0·3 while $C_2$ is 0·6. Launder (1975*a*) has found, however, that the effect of stratification on the nearly homogeneous free shear layer is predicted better by taking *both* $C_3$ *and* $C_2$ equal to about 0·6. The present computations suggest that the latter values are close to the optimum ones in near-wall flows too.

In a simple shear flow the proximity of a rigid wall modifies the pressure field, thus impeding the transfer of energy from the streamwise direction to that normal to the wall. The relative magnitude of the shear stress is also diminished. By introducing the unit vector $\mathbf{n}$ normal to the surface, Shir (1973) advanced the following near-wall addition to $\phi_{ij,1}$ producing generally the desired behaviour:

$$\phi'_{ij,1} = C'_1(\epsilon/k)\,(\overline{u_k u_m}\, n_k n_m \delta_{ij} - \tfrac{3}{2}\overline{u_k u_i}\, n_k n_j - \tfrac{3}{2}\overline{u_k u_j}\, n_k n_i) f(l/n_i r_i), \tag{14}$$

where $\mathbf{r}$ is the position vector and $l$ is a characteristic turbulence length scale.The factor $\frac{3}{2}$ is needed to render the tensor traceless. While Shir used (10) to approximate $\phi_{ij,2}$ he assumed that there was no corresponding modification due to the ground, a view which seems inconsistent with (8). Here we adopt the idea expressed by (14) and apply it also to the other components of $\phi_{ij}$:

$$\phi'_{ii,2} = C'_2(\phi_{km,2} n_k n_m \delta_{ij} - \tfrac{3}{2}\phi_{ik,2} n_k n_j - \tfrac{3}{2}\phi_{jk,2} n_k n_i) f(l/n_i r_i), \tag{15}$$

$$\phi'_{ij,3} = C'_3(\phi_{km,3} n_k n_m \delta_{ij} - \tfrac{3}{2}\phi_{ik,3} n_k n_j - \tfrac{3}{2}\delta_{jk,3} n_k n_i) f(l/n_i r_i). \tag{16}$$

The corresponding terms in the heat-flux equation are modelled in a precisely parallel way. Following Monin (1965), the first part of the pressure–temperature-gradient correlation is written as

$$\phi_{i\gamma,1} = -C_{1\gamma}(\epsilon/k)\,\overline{u_i \gamma}. \tag{17}$$

Since, in a locally isotropic thermal field, direct molecular destruction of $\overline{u_i \gamma}$ is negligible, it is $\phi_{i\gamma,1}$ which prevents the heat-flux correlations, generated by mean temperature gradients, from growing indefinitely. As in our earlier work, we assume that mean-strain and gravitational effects on the pressure field will act to diminish in magnitude the direct generation. Thus

$$\phi_{i\gamma,2} = -C_{2\gamma} P_{i\gamma}, \quad \phi_{i\gamma,3} = -C_{3\gamma} G_{i\gamma}. \tag{18), (19}$$

The wall-correction terms for these three processes are assumed to take a form corresponding to (14)–(16):

$$\phi'_{i\gamma,1} = -C'_{1\gamma}(\epsilon/k)\,\overline{u_k \gamma}\, n_i n_k f(l/n_i r_i), \tag{20}$$

$$\phi'_{i\gamma,2} = C'_{2\gamma} C_{2\gamma} P_{k\gamma} n_i n_k f(l/n_i r_i), \tag{21}$$

$$\phi'_{i\gamma,3} = C'_{3\gamma} C_{3\gamma} G_{k\gamma} n_i n_k f(l/n_i r_i). \tag{22}$$

We note that the near-wall function appearing in (20)–(22) is assumed to be the same as that in (14)–(16), or rather that the argument of the functions is the same in each case. A different length-scale variation could have been adopted for the stress and heat-flux field but there seemed insufficient experimental evidence to warrant such an elaboration. It is convenient to choose $f$ such that its value is unity close to the wall, where, in neutral flow, the length scale increases linearly with height; this choice essentially fixes the coefficients $C'_1$, etc. of the near-wall corrections. At this stage, apart from the choice of empirical coefficients and the wall-damping function, (1) and

(2) are closed except for the approximation of the mean-square temperature variance appearing in $G_{i\gamma}$ [see (6)]. In local equilibrium the exact transport equation for $\overline{\gamma^2}$ reduces to

$$-\overline{u_j\gamma}\,\partial\Gamma/\partial x_j = \epsilon_\gamma \equiv \lambda\overline{(\partial\gamma/\partial x_j)^2}, \tag{23}$$

from which $\overline{\gamma^2}$ is absent. The temperature variance may be re-introduced by way of the dimensionless quantity

$$R \equiv \frac{\epsilon}{\epsilon_\gamma}\frac{\overline{\gamma^2}}{2k}, \tag{24}$$

which represents the ratio of the characteristic decay times for the turbulent temperature and velocity fields. In decaying grid turbulence the equilibrium level of $R$ seems to be well established as unity (Newman, Launder & Lumley 1978; Warhaft & Lumley 1978). In shear flows close to local equilibrium there is some evidence that a smaller value of $R$ is appropriate. A value of about 0·5 is suggested by Dekeyser, Beguier & Launder (1976) based on a re-analysis of various shear-flow studies at the IMST, Marseille, while the present writers have used values of 0·7 and 0·8 in the prediction of buoyant free shear flows. On inserting (24) into (23) we obtain

$$\overline{\gamma^2} = -C_\gamma(k/\epsilon)\,\overline{u_j\gamma}\,\partial\Gamma/\partial x_j, \tag{25}$$

where the coefficient $C_\gamma$ has been introduced in place of $2R$.

## 3. Horizontal boundary-layer flow

Substitution of the above modelling assumptions into (1) and (2) and application of the boundary-layer approximations produces the following equation set for horizontal flow (the streamwise direction is taken as $x_1$ and $x_3$ is vertically upwards from the horizontal plane surface):

$$\frac{\overline{u_1^2}}{k} = \tfrac{2}{3}(1+\phi_4)+\phi_5\frac{\overline{u_3^2}}{k}+2\phi_3\frac{R_f}{1-R_f}, \tag{26}$$

$$\frac{\overline{u_2^2}}{k} = \tfrac{2}{3}(1-\phi_1)+\phi_5\frac{\overline{u_3^2}}{k}+2\phi_6\frac{R_f}{1-R_f}, \tag{27}$$

$$\frac{\overline{u_3^2}}{k} = \frac{2}{3}\frac{1-\phi_2}{1+2\phi_5}-\frac{2\phi_7}{1+2\phi_5}\frac{R_f}{1-R_f}, \tag{28}$$

$$-\frac{\overline{u_1u_3}}{k} = \phi\frac{k}{\epsilon}\frac{\overline{u_3^2}}{k}\frac{\partial U_1}{\partial x_3}-\phi'\frac{\alpha g}{\Gamma}\frac{\overline{u_1\gamma}}{\epsilon}, \tag{29}$$

$$-\overline{u_1\gamma} = \phi_\gamma\frac{k}{\epsilon}\overline{u_1u_3}\frac{\partial\Gamma}{\partial x_3}+\phi'_\gamma\frac{k}{\epsilon}\overline{u_3\gamma}\frac{\partial U_1}{\partial x_3}, \tag{30}$$

$$-\overline{u_3\gamma} = \phi_{\gamma1}\frac{k}{\epsilon}\overline{u_3^2}\frac{\partial\Gamma}{\partial x_3}+\phi'_{\gamma1}C_\gamma\frac{k^2}{\epsilon^2}\frac{\alpha g}{\Gamma}\overline{u_3\gamma}\frac{\partial\Gamma}{\partial x_3}, \tag{31}$$

where $R_f$ is the flux Richardson number:

$$R_f \equiv -G/P. \tag{32}$$

In arriving at the above forms use has been made of the fact that the total generation rate of turbulence energy equals the dissipation rate, i.e. $P+G = \epsilon$.

Denoting the length-scale function $f(l/x_3)$ by $f$ for brevity, the coefficients $\phi$ can be written in terms of the model constants as follows:

$$\phi_1 = (1 - C_2 - C_2' C_2 f)/C_1, \tag{33}$$

$$\phi_2 = (1 - C_2 + 2C_2' C_2 f)/C_1, \tag{34}$$

$$\phi_3 = (3 - 2C_2 - C_3 + C_2' C_2 f + 2C_3' C_3 f)/3C_1, \tag{35}$$

$$\phi_4 = (2 - 2C_2 + C_2' C_2 f)/C_1, \tag{36}$$

$$\phi_5 = C_1' f/C_1, \tag{37}$$

$$\phi_6 = (C_2 - C_3 + C_2' C_2 f + 2C_3' C_3 f)/3C_1, \tag{38}$$

$$\phi_7 = (3 - C_2 - 2C_3 + 2C_2' C_2 f + 4C_3' C_3 f)/3C_1, \tag{39}$$

$$\phi = (1 - C_2 + 1{\cdot}5C_2' C_2 f)/(C_1 + 1{\cdot}5C_1' f), \tag{40}$$

$$\phi' = (1 - C_3 + 1{\cdot}5C_3' C_3 f)/(C_1 + 1{\cdot}5C_1' f). \tag{41}$$

For the two heat-flux equations (30) and (31), the coefficients are

$$\phi_\gamma \equiv C_{1\gamma}^{-1}, \quad \phi_\gamma' = (1 - C_{2\gamma})\,\phi_\gamma, \tag{42), (43}$$

$$\phi_{\gamma 1} = [C_{1\gamma} + C_{1\gamma}' f]^{-1}, \quad \phi_{\gamma 1}' = (1 - C_{3\gamma} + C_{3\gamma}' C_{3\gamma} f)\,\phi_{\gamma 1}. \tag{44), (45}$$

We note that, owing to the form of (20)–(22), wall effects are absent from the equation for the horizontal heat flux and appear only in the equation for $\overline{u_3\gamma}$ through the part of the pressure correlation involving buoyancy. As a consequence the constant $C_{2\gamma}'$ in (21) disappears from the boundary-layer form of the model.

Equations (26)–(28) are straightforward expressions for the dimensionless normal stresses $\overline{u_i^2}/k$ in terms of the local flux Richardson number. Substitution for $\overline{u_i\gamma}$ from (30) in (29) yields a gradient-diffusion type of expression for the shear stress. The equivalent expression for the vertical heat flux is obtained directly by rearrangement of (31). Thus

$$-\overline{u_1 u_3} = \beta(k/\epsilon)\,\overline{u_3^2}\,\partial U_1/\partial x_3, \tag{46}$$

$$-\overline{u_3 \gamma} = \eta(k/\epsilon)\,\overline{u_3^2}\,\partial \Gamma/\partial x_3, \tag{47}$$

where

$$\beta \equiv \frac{\phi}{1 + \phi' B(\phi_\gamma + \phi_\gamma'/\sigma_t)}, \tag{48}$$

$$\eta \equiv \frac{\phi_{\gamma 1}}{1 + \phi_{\gamma 1}' C_\gamma B} \tag{49}$$

and $B$ is a dimensionless buoyancy parameter defined by

$$B \equiv \frac{k^2}{\epsilon^2}\frac{\alpha g}{\Gamma}\frac{\partial \Gamma}{\partial x_3}. \tag{50}$$

It is convenient to denote the ratio $K_M/K_H$ of the turbulent exchange coefficients by $\sigma_t$, the turbulent Prandtl number, which is obtained from (48) and (49) as

$$\sigma_t \equiv \frac{\beta}{\eta} = \frac{\phi}{\phi_{\gamma 1}}\frac{1 + \phi_{\gamma 1}' C_\gamma B}{1 + \phi' B(\phi_\gamma + \phi_\gamma'/\sigma_t)}. \tag{51}$$

A second expression relating $B$ to $\sigma_t$ is found by combining the definitions of $B$ and $R_f$ and eliminating the velocity gradient by means of the definition

$$P+G \equiv -\overline{u_1 u_3}(\partial U_1/\partial x_3)(1-R_f). \tag{52}$$

After some algebra there results

$$B = \frac{A\sigma_t}{\phi - \phi' A(\phi_\gamma \sigma_t + \phi'_\gamma)}, \tag{53}$$

in which

$$A \equiv \frac{k}{\overline{u_3^2}} \frac{R_f}{1-R_f}. \tag{54}$$

Combination of (50) and (51) then produces an explicit expression for $\sigma_t$:

$$\sigma_t = \frac{\phi - \phi'\phi'_\gamma A}{\phi_{\gamma 1} - (\phi'_{\gamma 1} C_\gamma - \phi'\phi_\gamma) A}. \tag{55}$$

Other useful quantities are then readily obtained as follows:

$$\left(-\frac{\overline{u_1 u_3}}{k}\right)^2 = \frac{\beta}{1-R_f} \frac{\overline{u_3^2}}{k}, \tag{56}$$

$$\frac{\overline{u_1 \gamma}}{\overline{u_3 \gamma}} = \frac{\overline{u_1 u_3}}{\overline{u_3^2}} \frac{\sigma_t \phi_\gamma + \phi'_\gamma}{\beta}, \tag{57}$$

$$(\overline{u_1 \gamma})^2/\overline{u_3^2}\,\overline{\gamma^2} = \eta/C_\gamma. \tag{58}$$

## 4. The choice of model constants

The proposed model for horizontal boundary layers contains twelve empirical coefficients to be chosen by reference to experimental data. Five of these are determined from measurements of unstratified free shear flows, two relate specifically to buoyant effects while the remaining five are introduced in modelling the wall-suppression factors. The 'unstratified' coefficients have been selected from laboratory experiments in flows where buoyant effects are entirely negligible. In some respects these data do not correspond precisely to those measured in the atmospheric boundary layer under neutral conditions; we shall discuss such differences later.

Table 2 gives the model constants used and some values of key turbulence quantities predicted for neutral free and wall flows in local equilibrium. Some small changes from our earlier proposals (Gibson & Launder 1976) may be noted. $C_2$ now takes the value 0·6 (instead of 0·55) required to satisfy (10) for isotropic turbulence and $C_1$ takes the lower value 1·8 (instead of 2·2) to improve slightly the predicted level of normal stresses in free shear flows. (This value is also in close agreement with that predicted by Herring (1974) from a direct-interaction calculation.)

$C_{1\gamma}$ and $C'_{1\gamma}$ are evaluated by reference to turbulent Prandtl number values typical of free and wall flow. For neutral conditions ($R_f = 0$) (55) reduces to

$$\sigma_{t0} = \frac{1 - C_2 + 1{\cdot}5 C'_2 C_2 f}{C_1 + 1{\cdot}5 C'_1 f}(C_{1\gamma} + C'_{1\gamma} f), \tag{59}$$

| Model constant | Value | Quantity | Basis for determination (neutral flow): Homogeneous shear layer | Near-wall turbulence |
|---|---|---|---|---|
| $C_1$ | 1·8 | $\overline{u_1^2}/k$ | 0·96 | 1·10 |
| $C_2$ | 0·6 | $\overline{u_2^2}/k$ | 0·52 | 0·65 |
| $C_1'$ | 0·5 | $\overline{u_3^2}/k$ | 0·52 | 0·25 |
| $C_2'$ | 0·3 | $\overline{u_1 u_3}/k$ | −0·34 | −0·26 |
| $C_{1\gamma}$ | 3·0 | $\sigma_t$ | 0·67 | 0·92 |
| $C_{1\gamma}'$ | 0·5 | | | |
| $C_{2\gamma}$ | 0·33 | $\overline{u_1\gamma}/\overline{u_3\gamma}$ | −1·3 | −2·1 |
| $C_3$ | 0·5 | Computer studies | | |
| $C_{3\gamma}$ | 0·33 | Isotropic turbulence | | |
| $C_3'$ | 0 | No information | | |
| $C_{3\gamma}'$ | 0 | No information | | |
| $C_\gamma$ | 1·6 | See discussion in text | | |

TABLE 2. Model constants and the basis for their determination.

where the function $f$ takes the values unity and zero in wall and free shear flows respectively.

Substitution of $C_{1\gamma} = 3{\cdot}0$ and $C_{1\gamma}' = 0{\cdot}5$ together with the other constants already specified produces values of $\sigma_{t0}$ of 0·67 and 0·92 for free and wall flow respectively. The quantity $C_\gamma$, which as we have noted is twice the time-scale ratio for the decay of temperature and velocity fluctuations, retains the value 1·6 proposed in Launder (1975*a*) and the four remaining constants $C_3$, $C_3'$, $C_{3\gamma}$ and $C_{3\gamma}'$ appear only for stratified flow.

For isotropic turbulence the quantities $C_3$ and $C_{3\gamma}$ (the coefficients in the buoyant parts of $\phi_{ij}$ and $\phi_{i\gamma}$) take the values 0·3 and $\frac{1}{3}$ respectively (Launder 1975*b*, 1976). It was our original intention to retain both these values in the present work. Distinctly better agreement was achieved, however, by choosing a higher value for $C_3$; this suggests that (despite the contrary indications from analysis of isotropic turbulence) in a shear flow the pressure fluctuations deflect or destroy buoyant generation about as effectively as they do shear generation. (This parity had in fact been assumed in our earlier work.) In tune with the above conclusion we have adopted the same value for $C_{2\gamma}$ as for $C_{3\gamma}$. The quantities $C_3'$ and $C_{3\gamma}'$ appear to exercise little influence in the prediction of stratified wall flows and, in the absence of definite information, they have been set to zero.

## 5. Surface-layer similarity and the length-scale function

According to the Monin–Oboukhov similarity hypothesis the turbulence structure in the uniform-stress layer close to the ground is determined solely by the quantities $-\overline{u_1 u_3}$, $-\overline{u_3\gamma}$, $\alpha g/\Gamma$ and $x_3$, which are combined in the single parameter

$$\zeta \equiv \frac{x_3}{L} = -\,Kx_3\frac{\alpha g}{\Gamma}\frac{\overline{u_3\gamma}}{u_\tau^3}, \tag{60}$$

where $K$ is the von Kármán constant, $u_\tau$ the 'friction velocity' $[\equiv(-\overline{u_1 u_3})^{\frac{1}{2}}]$ and $L$ the Monin–Oboukhov length scale defined by this equation. The hypothesis implies that, for flow at high Reynolds numbers in the surface layer, appropriately normalized turbulence correlations depend only on $\zeta$.

In the neutral surface layer $L \to \infty$, so that characteristic eddy sizes scale with $x_3$, $l$ varies linearly with height and the length-scale function $f$ is unity. This is the common mixing-length assumption for wall flow which has formed the basis of many successful prediction methods.

For conditions of strong stable stratification the eddy size is determined solely by the stability and not by distance from the surface. Clearly in this case the length-scale function $f$ tends to zero as in a free flow. The other asymptotic limit, represented by large negative Richardson numbers, is a condition of free convection with no predominant mean flow direction. The eddy sizes scale with $x_3$ and $f$ is theoretically unity. One important result of free-convection scaling may be noted here:

$$\overline{u_3^2}/u_\tau^2 \propto (-\zeta)^{\frac{2}{3}}. \tag{61}$$

The dependence of the length scale on stability between the asymptotic limits may be deduced from measurements of the mean velocity profile in the surface layer. If we define a scale typical of the energy-containing motion by

$$l \equiv (-\overline{u_1 u_3})^{\frac{3}{2}}/\epsilon \tag{62}$$

the condition of local equilibrium yields the following expression for the length-scale function:

$$f = l/Kx_3 = 1/\{\phi_m(1-R_f)\}, \tag{63}$$

where $\phi_m$ is the dimensionless mean shear defined by

$$\phi_m \equiv \frac{Kx_3}{u_\tau}\frac{\partial U_1}{\partial x_3}. \tag{64}$$

Numerous empirical expressions for $\phi_m$ have been quoted in the literature. For stably stratified flow there appears to be general agreement that the data may be fitted by a function of the form

$$\phi_m = (1+\beta_1\zeta) = (1-\beta_1 R_f)^{-1}. \tag{65}$$

Values of $\beta_1$ ranging from 7 to 10 are reported from wind-tunnel measurements by Arya & Plate (1969) while the atmospheric turbulence data reviewed by Busch (1972) suggested values ranging from 4·7 to 7·0. We have chosen $\beta_1 = 5{\cdot}5$ to specify $f$ in the present calculations. This somewhat arbitrary choice in the middle of the range quoted by Busch was made to produce optimum agreement with the stable-flow data although, in fact, the model predictions are not very sensitive to quite large changes in this quantity; values of 4·7 and 7·0 still produce acceptable results.

For unstable conditions it is expected that the eddy sizes scale approximately with $x_3$ and that the function $f$ does not depart greatly from unity. We have used the KEYPS formula (Panofsky 1963):

$$\phi_m = (1-\beta_2 R_f)^{-\frac{1}{4}}, \tag{66}$$

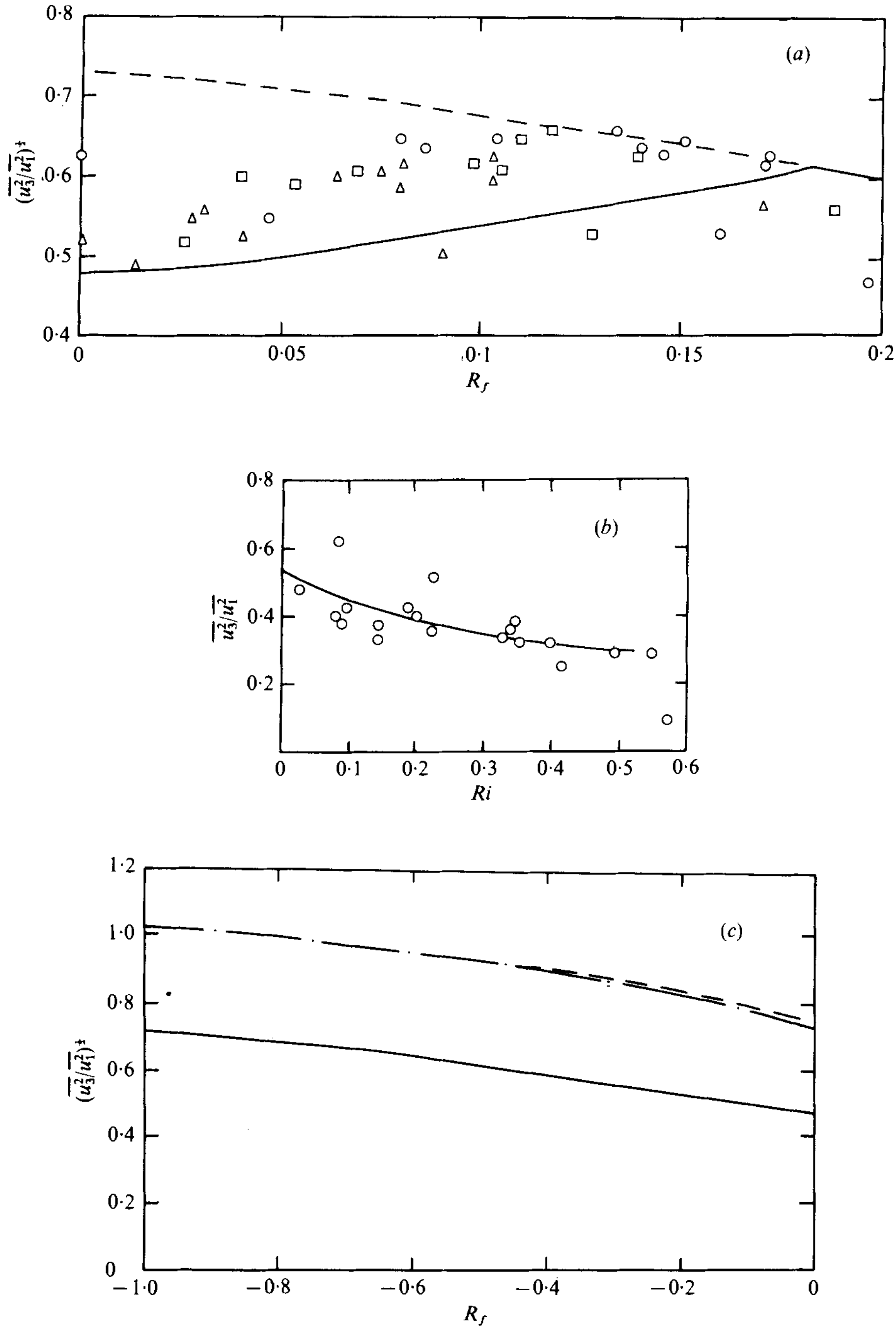


FIGURE 1. (*a*) Dependence of $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ on flux Richardson number in stably stratified flow. Atmospheric-boundary-layer data from Haugen *et al.* (1971). △, $x_3 = 5{\cdot}7$ m; □, $x_3 = 11{\cdot}4$ m; ○, $x_3 = 22{\cdot}6$ m. Predictions: ——, wall flow; – – –, free shear flow. (*b*) Dependence of the normal-stress ratio on Richardson number in a stably stratified free shear flow. Data from Young (1975). (*c*) Predicted dependence of the normal-stress ratio on Richardson number in unstably stratified flow. ——, wall flow; –·–·–, free shear flow ($P+G = \epsilon$); – – –, free shear flow ($P+G = 0{\cdot}8\epsilon$).

with $\beta_2 = 14$ in accord with the data reviewed by Busch (1972). Combination of (65) and (66) with (63) produces the following expressions for $f$:

$$f = \frac{l}{Kx_3} = \begin{cases} \dfrac{1-5\cdot 5R_f}{1-R_f} & \text{for} \quad R_f > 0, \quad (67a) \\ \dfrac{(1-14R_f)^{\frac{1}{4}}}{1-R_f} & \text{for} \quad R_f < 0. \quad (67b) \end{cases}$$

It should be noted that the assumptions of this section are not an integral feature of the turbulence model. In a complete second-order closure the length-scale variation could, for example, be calculated by solving a transport equation for the energy dissipation rate $\epsilon$. A well-tested form for this equation in stratified flow, however, does not yet seem to be available. A directly prescribed length-scale equation seemed, therefore, the best way of testing the basic conjectures of the present paper.

## 6. Presentation and discussion of results

In this section we compare the predicted and measured influence of stratification on the atmospheric boundary layer and nearly homogeneous free shear flow. The experimental sources are the Kansas measurements in the lower 25 m of the atmospheric boundary layer and Young's (1975) study of free shear flow, which was undertaken in an improved version of Webster's (1964) tunnel. The latter results are similar in many respects to Webster's original values though they display better internal consistency. Our main attention is given to the atmospheric boundary layer because the free shear layer has been specifically studied in an earlier paper (Launder 1975*a*). Nevertheless comparisons for the free shear flow are included to bring out the dramatically different behaviour exhibited by the two types of flow.

Figures 1 (*a*)–(*c*) show some of the most significant differences between the near-wall and the free flow. Under stable conditions the predicted level of $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ falls smoothly for the free shear layer and rises for the boundary layer as $R_f$ is increased until, for a flux Richardson number of about 0·18, the relative stress levels in the two flows are equal. The reason is that the function $f$ given by (67*a*) has then fallen to zero so no wall effect is felt. This is what is indicated in figure 1 (*a*). The experimental data in this figure are those assembled by Arya (1972) from the Kansas measurements which were obtained at heights of 5·7, 11·4 and 22·6 m. Although the experimental points are scattered the definite impression is conveyed that the ratio of vertical to streamwise velocity fluctuations does indeed rise in a similar fashion to the predicted variation, though the measured values lie about 15 % higher than the calculated ones. We have not included in this figure Arya's own data obtained in the stratified wind tunnel at Colorado State University. These, in fact, show two patterns of variation, $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ increasing very steeply with $R_f$ for flux Richardson numbers below 0·02 while displaying a behaviour like the atmospheric data over the range $0{\cdot}025 < R_f < 0{\cdot}06$. If they had been included in figure 1(*a*) the impression that Arya's measurements would have given to the figure as a whole would have been that of a large amount of scatter with *no* ordered dependence on $R_f$.† It is probably for this reason that no one has hitherto remarked on the 'unnatural' behaviour displayed by the atmospheric data.

The atmospheric measurements do perhaps indicate a somewhat steeper rise of $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ with Richardson number than do the predictions and a maximum level that is reached at a lower value of $R_f$. Both these results suggest that the chosen value of $\beta_1$ may be 20% too small. However, such details do not affect the main result to emerge: namely, that inserting the experimentally observed dependence of length scale on buoyancy into a wall-effect model of the pressure–strain correlation (devised for *non*-stratified flows) gives broadly the correct dependence of $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ on $R_f$. This result, which is in marked contrast with the behaviour predicted by all existing closures known to the authors, provides strong support for the main concepts of the present model.

Predictions for a free shear flow in local equilibrium are made by substituting $f = 0$ in (33)–(45). They are compared with Young's (1975) measurements in figure 1(*b*). Note that to avoid errors associated with reprocessing the data we have retained from the original report the normal-stress ratio (rather than its square root) and, in place of the flux Richardson number, the gradient Richardson number defined by

$$R_i \equiv \frac{\alpha g}{\Gamma} \frac{\partial \Gamma / \partial x_3}{(\partial U_1 / \partial x_3)^2} = \sigma_t R_f. \tag{68}$$

The data show that the normal-stress ratio decreases with increasing stable stratification as is predicted by the model. For *unstable* stratification, figure 1(*c*) indicates that the present model gives $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ increasing with $|R_f|$ for both the boundary layer and the free flow. This is due to the fact that the length scale in unstable conditions does not depart far from proportionality with distance from the surface and the wall effect is not appreciably affected by the degree of stratification.

The predicted variation of the r.m.s. vertical velocity fluctuations (normalized by the friction velocity) under unstable conditions is compared with the measurements of Wyngaard *et al.* (1971) in figure 2. The correct trend is displayed by the predictions but they lie about 20% below the measured values. The difference appears to be due mainly to too small values of the shear stress being measured in the Kansas study (the implied value of the von Kármán constant was only 0·35 compared with more usually reported values of about 0·41). Certainly most laboratory studies have found, in line with the present prediction, that under neutral conditions $(\overline{u_3^2})^{\frac{1}{2}}/u_\tau$ is very close to unity. For strongly unstable conditions ($-x_3/L > 2{\cdot}0$), the calculations exhibit a $\frac{1}{3}$-power dependence which is substantially in agreement with the measured behaviour and the similarity hypothesis for free convection [equation (61)].

† There are a number of possible reasons for Arya's near-wall measurements exhibiting a different trend from those in the atmosphere. The data taken at 1% of the boundary-layer thickness may well be subject to viscous effects (the streamwise r.m.s. component very near a smooth wall displays marked peaking, a feature absent in the boundary layer on a rough wall; Pimenta, Moffat & Kays 1975). At the next position at which data are reported (5% of the boundary-layer height) the shear stress studied appears to differ significantly from wall values; this suggests that the structure may be some way from local equilibrium. It is unfortunately not possible to identify from the reported values of $(\overline{u_3^2}/\overline{u_1^2})^{\frac{1}{2}}$ the positions and velocities corresponding to each point. A further possible source of differences between the wind-tunnel and atmospheric boundary layers is that the thickness of the former was 40% of the tunnel width and thus almost certainly modified by the presence of the side walls of the test section.

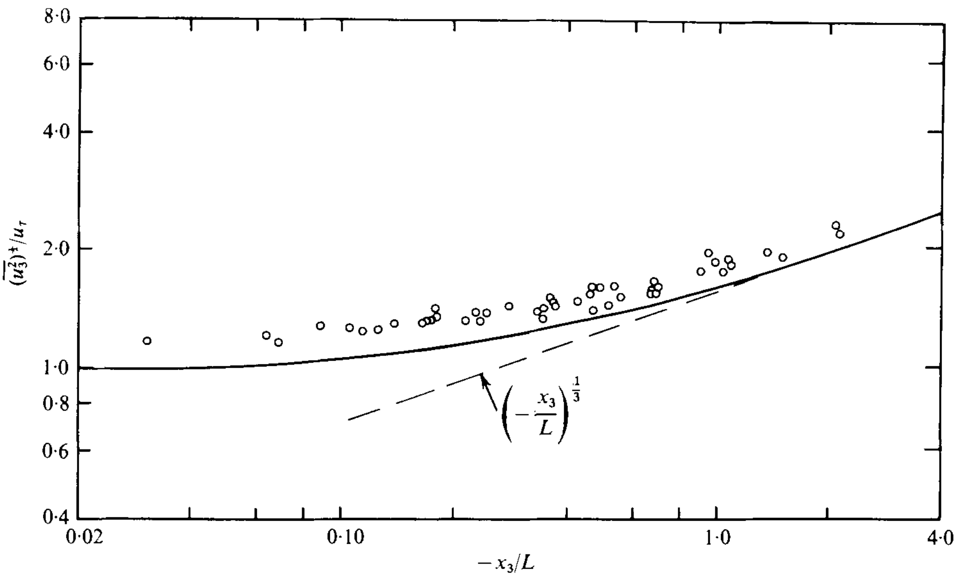


FIGURE 2. Measured and predicted r.m.s. vertical velocity fluctuations under unstable conditions. Atmospheric-boundary-layer data from Wyngaard *et al.* (1971).

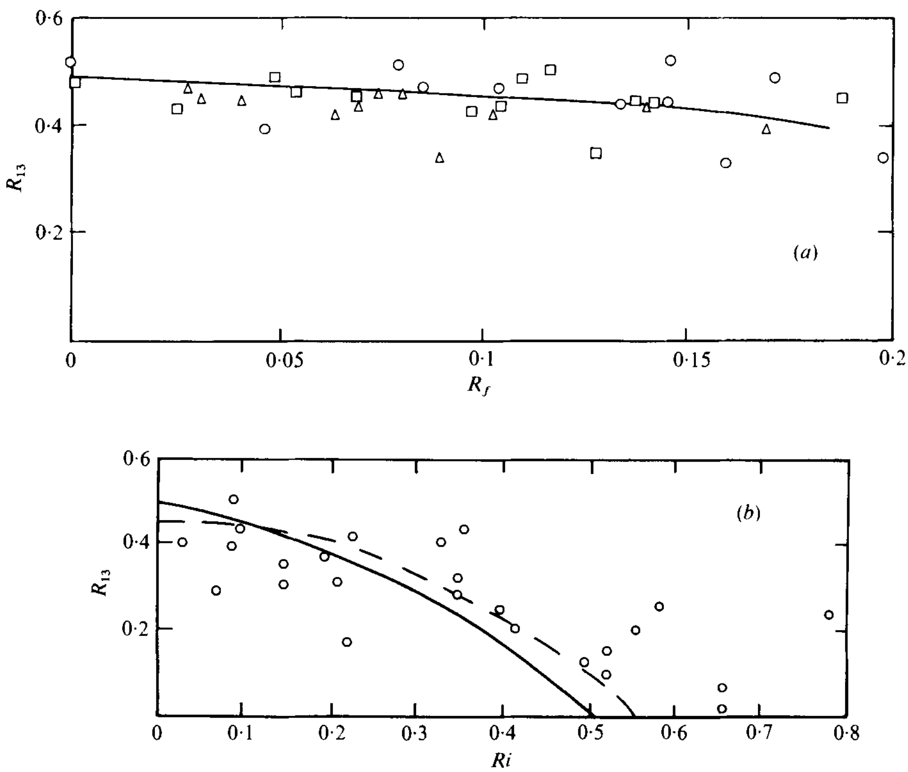


FIGURE 3. (*a*) Measured and predicted behaviour of the shear-stress correlation coefficient in stable flow. Atmospheric-boundary-layer data from Haugen *et al.* (1971). (*b*) Measured and predicted behaviour of the shear-stress correlation coefficient with increasing stability in a free shear flow. Data from Young (1975). Broken line denotes predictions for an energy production-to-dissipation ratio of 0·8.

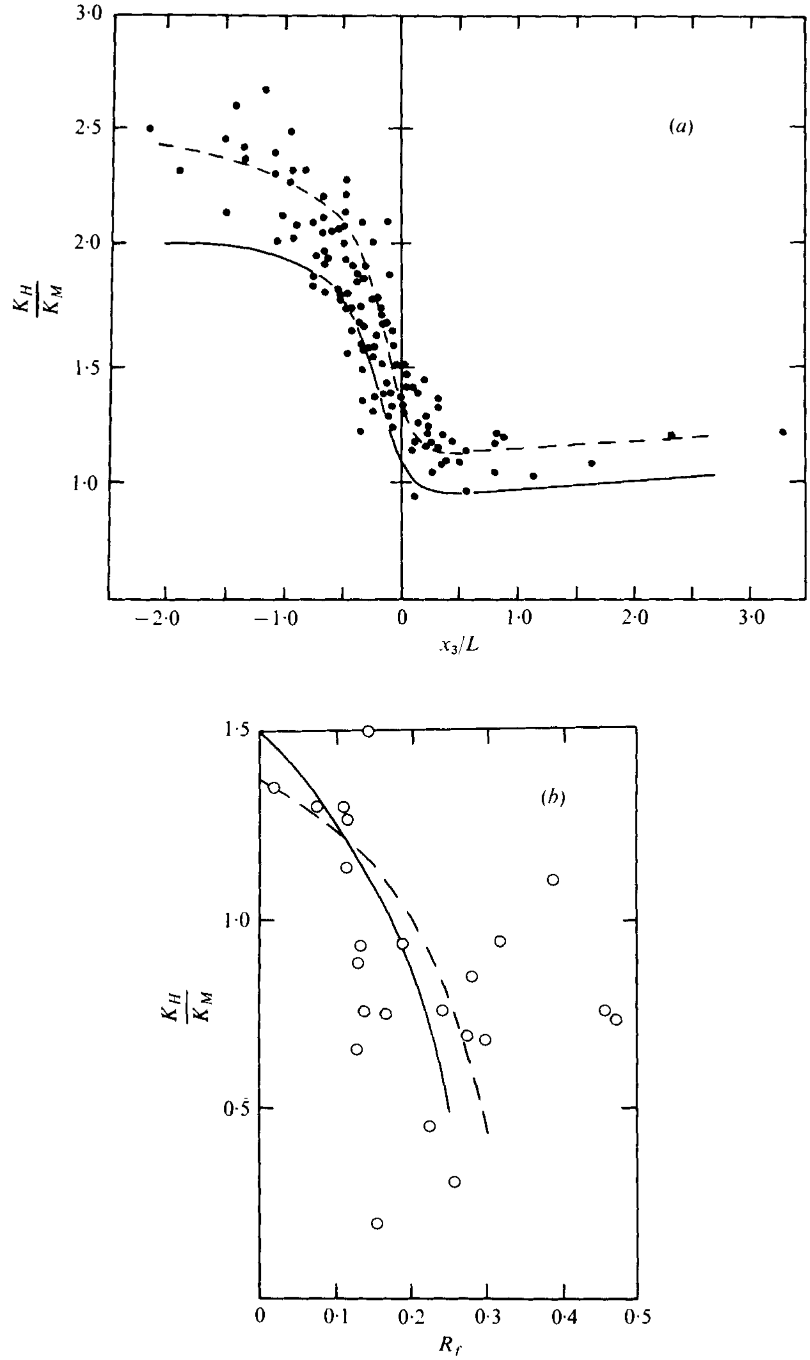


FIGURE 4. (a) Measured and predicted dependence on stability of the ratio of the turbulent exchange coefficients. Atmospheric-boundary-layer data from Businger *et al.* (1971). Broken line denotes 'adjusted' predictions described in the text. (b) Measured and predicted behaviour of the exchange-coefficient ratio with increasing stability in a free shear flow. Data from Young (1975). Broken line denotes predictions for an energy production-to-dissipation ratio of 0·8.

The final flow-field comparison in figures 3(*a*) and (*b*) shows the dependence of the shear-stress correlation coefficient on Richardson number for stable stratification. The correlation coefficients for the atmospheric boundary layer have been increased by 50 % to bring their level under near-neutral conditions more into line with isothermal laboratory studies.† The predictions support the observation made earlier that the correlation coefficient is relatively unaffected by buoyancy. The calculations do in fact display a slow decrease in $R_{13}$ as $R_f$ is increased, a variation which cannot be said to be displayed by experiment. The scatter in the data is so large, however, that the calculated variation is at least not inconsistent with the available measurements. The behaviour of this correlation coefficient for the free shear flow is virtually identical to that for the wall flow; this reflects the fact that in neutral flows $R_{13}$ is virtually the same in both wall and free flows so the near-wall correction has an insignificant effect on this parameter. The *apparent* sensitivity to buoyancy is, however, different because the data shown in figure 3(*b*) extend to higher values of the Richardson number. The calculated variation shows a decay similar to, but somewhat steeper than, Young's (1975) measurements; indeed $R_{13}$ falls to zero for $Ri$ just greater than 0·5, which corresponds to a critical flux Richardson number of 0·25. It is unlikely that in experiments on such strongly stratified flows local equilibrium will be maintained: the energy-containing motions will tend to be damped under the action of gravity with passage downstream but it seems unlikely that there would be a sufficient development length for these effects to be transferred through the spectrum to the fine-scale motion. It seems likely, therefore, that the ratio of the generation to the dissipation rate will fall somewhat below unity under strongly stable conditions. To illustrate the possible significance of such an effect figure 3(*b*) contains numerical results for a case where the total energy production-to-dissipation ratio is only 0·8, instead of unity. The governing equations, which are not presented here, are obtained by following precisely the analysis of §§ 2 and 3 with $\epsilon$ replaced by $1{\cdot}25(P+G)$ rather than by $P+G$. In this case the general trend is the same as before though the collapse of the shear-stress correlation coefficient is deferred.

Turning now to the properties of the temperature field, the ratio of the effective diffusivities of heat and momentum (the reciprocal of the turbulent Prandtl number) is shown in figures 4(*a*) and (*b*). Under neutral conditions the data for the atmospheric boundary layer indicate a value of 0·74 for the turbulent Prandtl number compared with the usually reported value of about 0·93 (Bradshaw 1976, p. 246). The difference appears to be partly due to the measured shear-stress levels being too low, a feature on which we have commented above. Two lines representing predicted behaviour are shown in figure 4(*a*): that directly given by the present model and the curve that results when the model values are increased by 20 % to bring them into line with the experimental data under neutral conditions. The prediction thus adjusted reproduces extremely closely the variations recorded in the experiment, including the decrease in the diffusivity ratio in weakly unstable flow and the approach to an asymptotic level under very strongly unstable conditions.

The overall agreement with experiment displayed by the present model under strongly stable conditions seems to confirm that gravity waves are not contributing significantly to measured 'turbulence' signals under the conditions studied. If they

† The fractional weighting was chosen as being a round number of about the correct magnitude.

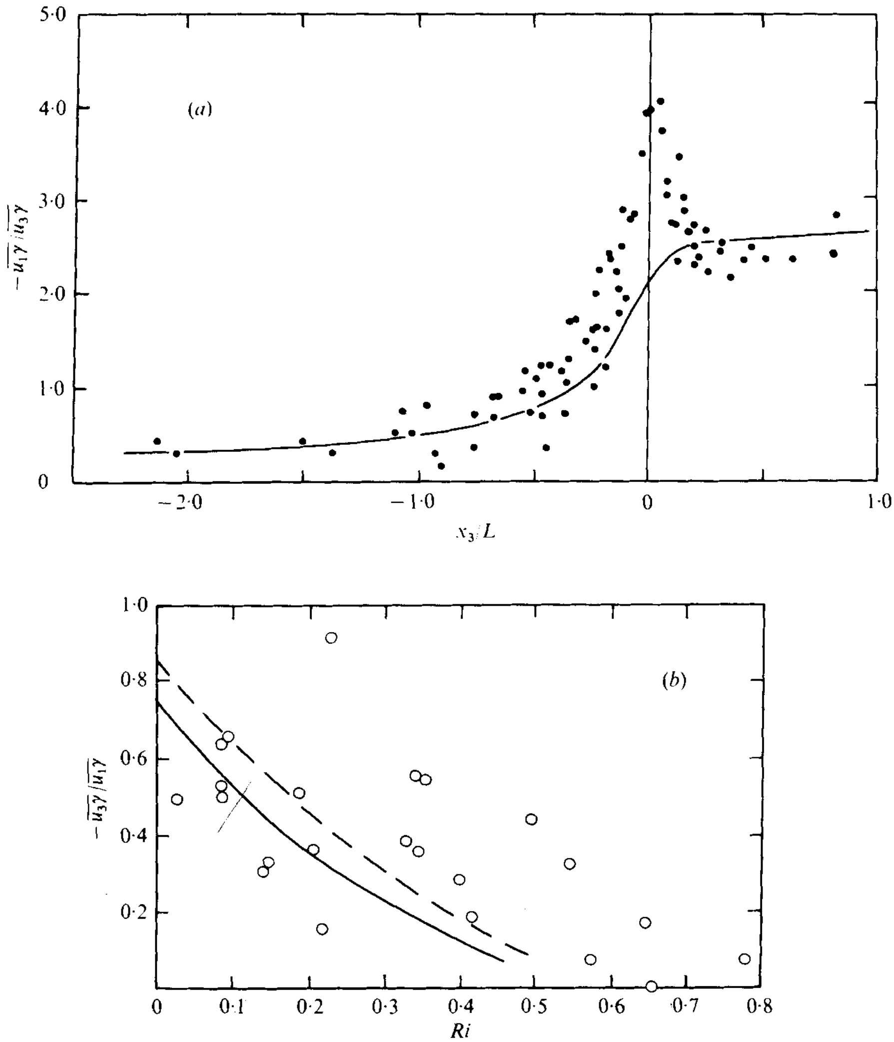


FIGURE 5. (*a*) Measured and predicted dependence on stability of the heat-flux ratio. Atmospheric-boundary-layer data from Wyngaard *et al.* (1971). (*b*) Measured and predicted behaviour of the heat-flux ratio with increasing stability in a free shear flow. Data from Young (1975). Broken line denotes predictions for an energy production-to-dissipation ratio of 0·8.

were, it is virtually certain that some properties of either the heat-flux or the stress field would be quite wrongly predicted by the present statistical approach.

The predicted behaviour in the homogeneous free shear flow is similar to that for the atmospheric boundary layer for unstable flows but is quite different under stable stratification. The ratio of diffusivities falls from a value of 1·5 for a neutral flow to approximately 0·5 for a flux Richardson number of 0·25. Young's (1975) data show a great deal of scatter but, like Webster's (1964) earlier experiments, support the trend displayed by the present prediction.

The ratio of horizontal to vertical heat flux is compared with the atmospheric data in figure 5(*a*). While the predicted behaviour in strongly stable or unstable stratification agrees satisfactorily with measurements, the measured strong peaking under neutral and very weakly stratified conditions is not reproduced. This is the first and only structural parameter for which the model displays a significantly different trend from the measurements. The value of the near-wall heat-flux ratio in neutral conditions is the subject of considerable uncertainty. Most of the values obtained in the laboratory under closely controlled conditions lie between 2·0 and 3·0 compared with nearly 4·0 for the data of figure 5. The value of just over 2·0 produced by the present model is a consequence of tuning the constants $C_\gamma$ and $C'_\gamma$ to fit the extensive measurements by Pimenta *et al.* (1975) on a uniformly roughened test plate. These data show little scatter and good internal consistency; overall we felt that they provided the best available set of thermal turbulence measurements in a boundary layer under neutral conditions. It does not seem possible to identify with certainty the cause of the different trend in the predicted and measured behaviour for $0 < x_3/L < 0{\cdot}5$. The particular choice of the length-scale dependence certainly affects the model and, in the course of optimizing the choice of length scale and coefficient in the present work, it was found that sometimes a mild diminution in $-\overline{u_1\gamma}/\overline{u_3\gamma}$ was predicted for $x_3/L > 0{\cdot}1$; these cases did not produce the best *overall* agreement, however. In any event, because the measured heat-flux ratios for neutral conditions shown in figure 5(*a*) are so much larger than values obtained in the laboratory it is open to question whether one should attempt to reproduce even the qualitative trend near $x_3/L = 0$. In this region the level of temperature fluctuations is necessarily very small and the signal contamination by velocity fluctuations may become significant.† In a free shear flow, for which results are shown in figure 5(*b*), there is a steady decrease in the ratio of vertical to horizontal heat flux. The predictions follow the trend of Young's (1975) measurements, though exhibiting a somewhat more rapid decrease. Again for illustration, the behaviour for a production-to-dissipation ratio of 0·8 is shown; for this case the rate of diminution is reduced.

## 7. Concluding remarks

The present study has, we feel, established that the strikingly different behaviour observed in the atmospheric boundary layer and the stratified shear layer is due to the sensitivity of the fluctuating pressure field to the ratio of a typical eddy size to the height above the ground. Stable stratification reduces the eddy size at a given height and thus diminishes the tendency of the ground to transfer velocity fluctuations and heat fluxes from the vertical to the horizontal direction.

The second-order model proposed in this study generally predicts the contrasting behaviour in the two kinds of flows with good accuracy. An exception is the strong peaking of the horizontal-to-vertical heat-flux ratio observed in the atmospheric boundary layer under nearly neutral conditions, a phenomenon that is not reproduced by the present closure. There are a number of simplifications in the model which may be responsible for the discrepancy, notably that the wall-effect function is the same for both the stresses and the heat fluxes. However, in view of the fact that, under neutral conditions, laboratory data generally exhibit much lower ratios of streamwise to normal heat fluxes than the atmospheric boundary layer, it is not entirely certain that the discrepancy is due to shortcomings of the model.

† In the paper reporting these measurements Wyngaard *et al.* (1971) have suggested that the apparent cusp near $x_3/L$ might reflect the effect of measuring heat fluxes in unsteady conditions.

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